1. Let $A, B$ be $3 \times 3$ matrices and suppose $\det(A) = 3$ and $\det(B) = 10$. Find $\det(2AB^{-1})$.

2. Consider the matrix

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \end{bmatrix}.$$  

(a) Find all eigenvalues of $A$.

(b) Find the eigenspace corresponding to each eigenvalue found in part (a).

3. (a) Show that the set $S = \{p(x) \in \mathcal{P}_3 \mid p(0) = 0, p(1) = 0\}$ is a subspace of $\mathcal{P}_3$.

(b) Find a basis for $S$.

(c) What is the dimension of $S$?

4. Let $U$ and $W$ be subspaces of a vector space $V$ such that $U \cap W = \{0\}$. Let $u \in U$ and $w \in W$ and suppose that $u \neq 0$ and $w \neq 0$. Prove that $\{u, w\}$ is linearly independent.

5. Determine whether each of the following statements is true or false, give a brief justification of your answer.

(a) If $A$ and $B$ are $n \times n$ matrices then $\det(A + B) = \det(A) + \det(B)$.

(b) If $S = \{v_1, ..., v_n\}$ is a linearly dependent set of vectors, then any vector in $S$ can be expressed as a linear combination of the other vectors.

(c) If an $n \times n$ matrix is invertible, then the column vectors of the matrix form a basis for $\mathbb{R}^n$.

(d) If $W$ is a subspace of a finite-dimensional vector space $V$ then $\dim(W) \leq \dim(V)$. 