1. Consider the matrix
\[
A = \begin{bmatrix}
1 & 0 & 0 & 1 \\
0 & 5 & -4 & 2 \\
-1 & 0 & 3 & -1 \\
-2 & 0 & 0 & 1
\end{bmatrix}
\]
(a) Find \(\det(A)\).
(b) Is \(A\) invertible?

2. Consider the matrix
\[
A = \begin{bmatrix}
1 & 1 & 0 & 1 \\
2 & 3 & 0 & 4 \\
-1 & 1 & 0 & 3
\end{bmatrix}
\]
(a) Find a basis for the row space of \(A\).
(b) Find a basis for the column space of \(A\).
(c) Find \(\text{rank}(A)\).

3. Suppose that the set \(\{v_1, v_2, v_3\}\) is a basis for a vector space \(V\). Show that the set \(\{v_1, v_2, v_1 + v_3\}\) is also a basis for \(V\).

4. Let \(A\) be an \(n \times n\) matrix, prove that 0 is an eigenvalue of \(A\) if and only if \(\text{rank}(A) < n\).

5. Determine whether each of the following statements is true or false, give a brief justification of your answer.
(a) If \(A\) is an \(n \times n\) matrix, then \(\det(cA) = c \det(A)\).
(b) The intersection of any two subspaces of a vector space \(V\) is a subspace of \(V\).
(c) If \(V\) is a finite dimensional vector space, then any set of vectors that spans \(V\) is linearly independent.
(d) Any subset of a vector space containing the zero vector is linearly dependent.